

COLOUR by Dinesh Padiyar

Eye Sensitivity

Colour is perceived in the eye due to three types of cones in the retina. These cones have photo-receptor proteins called photopsins which are each sensitive to a different part of the spectrum. The convention in physiology is to call them L, M and S (Long, Medium, Short). The L cones are sensitive to a range from about 500nm to about 650nm and code for “red”, the M cones go from 450nm to about 600nm and code for “green” and the S from 400nm to 500nm for “blue”. Fig 1 shows sensitivity of the three cones. Note the overlap between the cones; an important point of tri-stimulus matching.

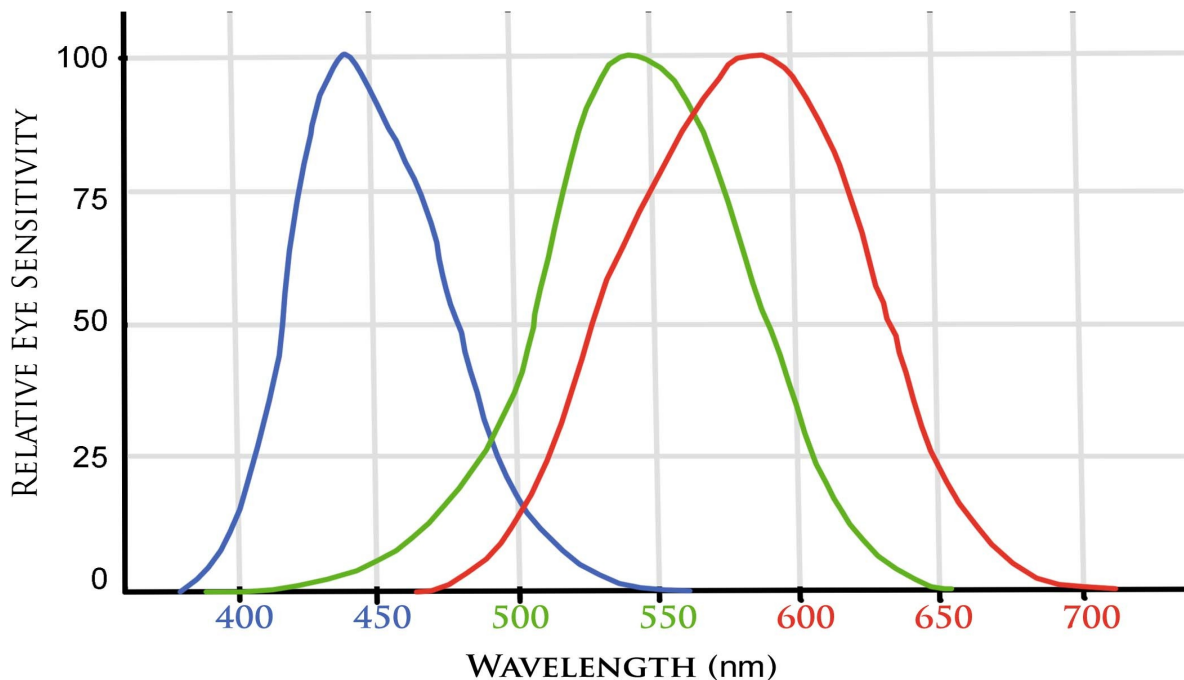


Fig 1. Sensitivity of cones

Activation of any one of these cones gives the appropriate colour response. Absence (or a weak response) of one or more of the cones will result in colour blindness where specific colour(s) are altered from normal viewing due to the absence of cone sensitivity in that range. Thus an absence of the L (“red”) cone will make all colours from 550 up to appear as green. In the range 550 - 475 (green-blue) will appear yellowish. Obviously, colour blindness is a very complex issue and this very simple explanation is only to illustrate the fact that cone activation results in the complex colours of normal viewing.

Note the overlap between cone activation of the three cones. Due to the overlap, it’s very difficult to simply activate only one cone to give only one colour response. Thus, while for any

light greater than 650nm or less than 420 only one cone is activated, this is an insignificant part of the visual spectrum. For the bulk of the visible spectrum any single wavelength will activate multiple cones at varying levels. So over a very wide range of wavelengths it's not possible to find a three-colour set of wavelengths or filters which will reproduce all colours by only stimulating just the red, green or blue cones alone without stimulating the cones of another colour.

Tri-Stimulus Matching

Colour reproduction is made possible by Grassman's laws, which state that any two or more stimuli that activate the different cones in the same way is perceived as the same colour regardless of spectral composition. Thus a stimulus composed of a combination of three certain primaries with different spectral powers, may be viewed as identical to a specific observed colour if the primaries stimulate the cones in an identical manner as the observed colour, even though the spectral composition (the quantity and specific colour breakdown) of the primaries is different to the colour to be matched. Such stimuli are known as *metameric pairs*, or *metamers*.

To see how this works, consider the cone response curves of fig 1 superimposed with three primaries of wavelengths 650 (Red), 530 (Green) and 460 (Blue) and a test colour to be matched of 600 (fig 2 adapted from "The Reproduction of Colour" by R.W.G. Hunt). The test colour of 600 is to be matched with two or more of the three primaries and the hope is that a combination of the primaries can be found, which, when summed, will activate the same cones as the test colour. If so, Grassman's Law tells us that despite the fact the sum of the primaries has a different spectral composition (two or three wavelengths of varying quantities) to the test colour (one wavelength), if the same cones are activated, the same colour will be seen.

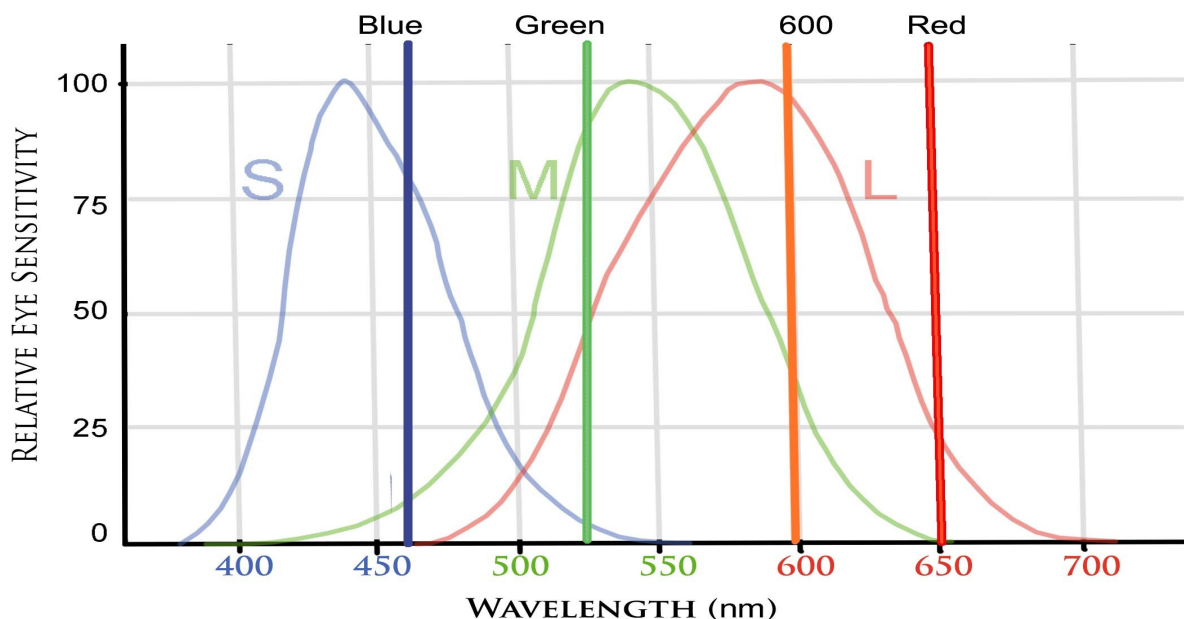


Fig 2 Primaries with test colour of 600

Table 1 shows the amounts by which the primaries activate the Red, Green and Blue cones as well as the amount by which the test colour of 600 activates the cones. If some combination of the primaries creates the same level of activation as the 600 alone, then the 600 will be matched by that combination, and will appear to be identical. The combination of primaries and the 600 will be a metameric pair. These numbers can be read right off the plot in fig 2

	Amount of Red Cone activation	Amount of Green Cone activation	Amount Blue Cone activation
Red Primary (650)	24	0	0
Green Primary (530)	56	92	4
Blue Primary (460)	0	9	75
600 (colour to be matched)	95	25	0

Table 1 Cone activation by three primaries and a test colour

As can be seen, the test colour of 600 activates the cones in the following way:

$$600 = 95 \text{ (Red cone)} + 25 \text{ (Green cone)} + 0 \text{ (Blue cone)}$$

To reconstruct this colour of 600, it's necessary to stimulate the three cones in the retina with these values. Reading across the chart, 0.27 of the green primary (530) results a value of 15 for red cone activation, 25 for green cone activation and 1 for blue cone activation. This gives the correct value for green, but is too low for red. A value of red cone activation of 95 is needed and the green primary only provides a value of 15. To get that extra 80 units of red, 3.3 of the red primary can be added, which gives 80 units of red cone activation only. So:

$$\begin{aligned}
 600 &= 3.33 \text{ (Red Primary)} + 0.27 \text{ (Green Primary)} + 0 \text{ (Blue Primary)} \\
 &= 95 \text{ units of red cone activation} + \\
 &\quad 25 \text{ units of green cone activation} + \\
 &\quad 1 \text{ unit of blue cone activation.}
 \end{aligned}$$

This is close enough to the correct activation for the 600 from a sum of certain fractions of the primaries. Therefore, this particular sum of these particular primaries will appear the same as the test colour of 600.

Can all colours be reproduced by these primaries? To determine this, a similar calculation will be carried out for a different test colour - 490nm. Fig 3 (taken from "The Reproduction of Colour" by R.W.G. Hunt) shows the same cone activation curves of fig 1 and the same three primaries, but with the 490 test colour superimposed

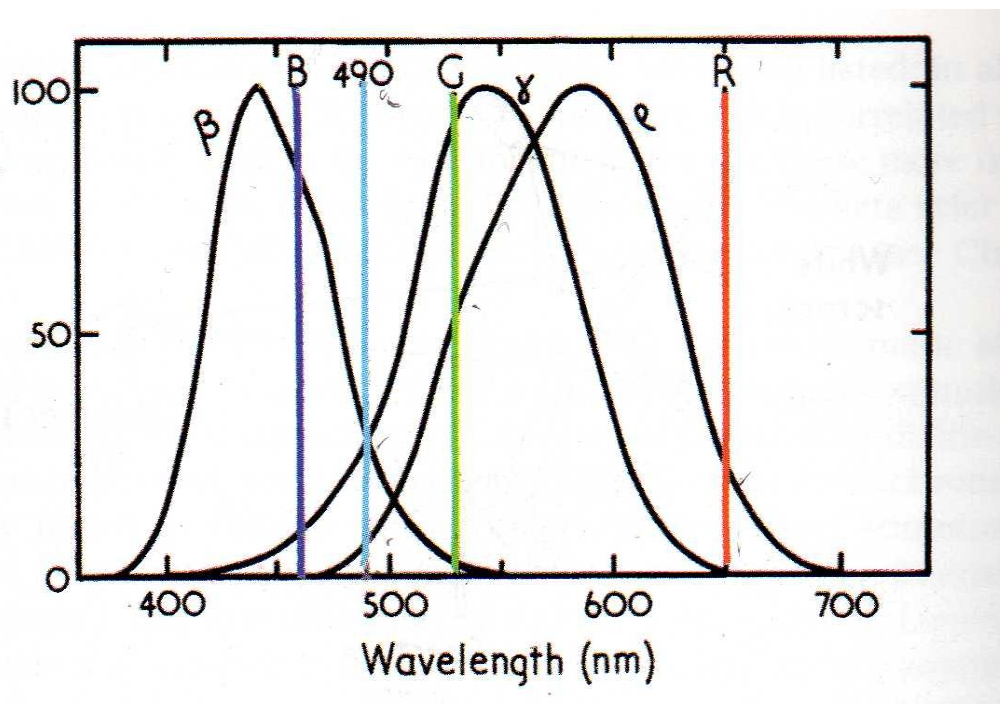


Fig 3 A test colour of 490 and three primaries of 650, 530 and 460

Table 1 is repeated but with the cone activation by 490:

	Amount of Red Cone activation	Amount of Green Cone activation	Amount Blue Cone activation
R primary	24	0	0
G Primary	56	92	4
B Primary	0	9	75
490	8	26	26

This time the test colour of 490 activates the cones:

$$490 = 8(\text{red cone}) + 26(\text{green cone}) + 26(\text{blue cone})$$

In order for the three primaries to activate the cones in the same way as the test colour, the nearest that can be achieved is to mix 0.3 of the B primary and 0.25 of the G. This then gives:

$$490 = ? \ 0.3 \ B + 0.25 \ G = 14(\text{red cone}) + 26(\text{green cone}) + 26(\text{blue cone})$$

Now, it can be seen that a mixture of these proportions of the primaries results in the correct

activation of two of the cones, the green and blue, but not the third. There is too much activation of the red sensitive cone. The mixture will appear more red shifted than the test colour. Adding any of the red primary will not only increase the red cone, of which there's already too much, but will also increase the green and blue cone. The mixture will look more purple. However, if a certain amount of red were to be added to the test colour, so that the test colour were made more red, then the primaries would match the *altered* test colour.

$$490 + 0.25 \text{ red primary} = 14L (\text{red cone}) + 26 (\text{green cone}) + 26 (\text{blue cone})$$

Thus an exact match is only accomplished by adding an amount of red to the test colour. This addition of the red primary to match the given test colour can be regarded, from an algebraic point of view, as an addition of a negative amount of red:

$$490 = 0.3 \text{ blue primary} + 0.25 \text{ green primary} - 0.25 \text{ red primary.}$$

The numbers representing the proportions of the primaries needed to match the test colour (+0.3, +0.25 and -0.25 in this case), are known as the *coefficients* of the primary, and, as can be seen in the above example, some of these coefficients are negative. The meaning of negative coefficients is that the appropriate amount of the primary (0.25 of the red primary, in this case) needs to be added to the test colour for a match. While it would appear that the colour of 490 cannot be exactly matched, the question of negative coefficients for a broader range of colours is taken up later on.

A similar calculation can be carried out for any colour in the visible in order to determine what proportions of the given primaries to add to stimulate the various cones in an identical manner. Thus, given a specific wavelength, the certain proportion of the three primaries - the coefficients - can be calculated as in the above examples to give an identical response as that specific wavelength. When these calculations are carried out for all colours in the visible spectrum it will result in three plots - one for the red primary, one for the green and one for the blue. These curves are known as the *colour matching function* curves. These curves can then be used to read off the proportions of the primaries - the coefficients - for any given wavelength by drawing a vertical line through the appropriate wavelength and noting where this line cuts the curves for the proportions of the red, green and blue primaries to match that wavelength.

The colour matching functions for the three primaries used here, 650, 530 and 460 are shown in fig 4 (From "The Reproduction of Colour" by R.W.G. Hunt).

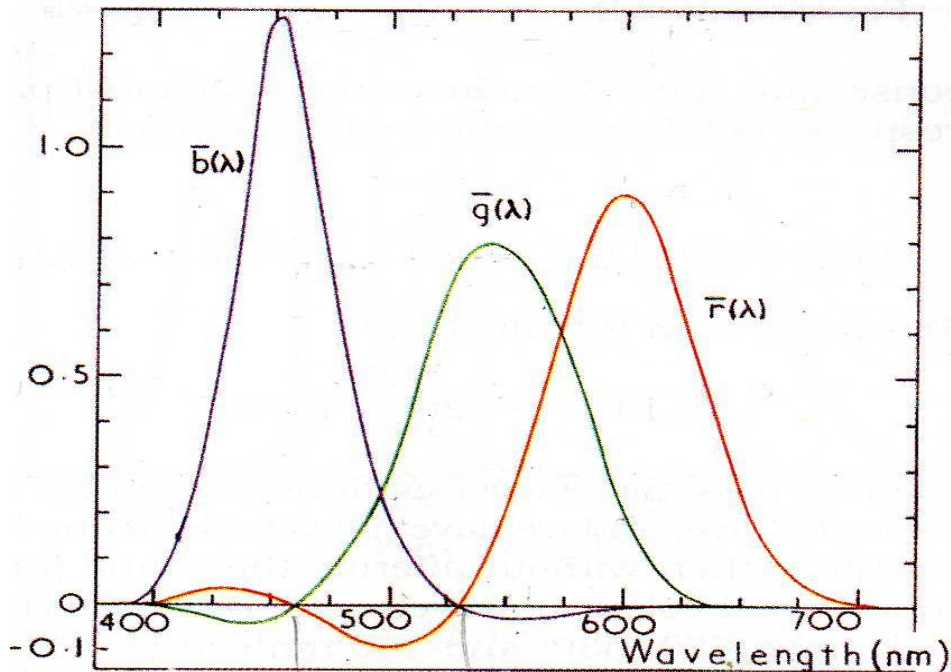


Fig 4 Colour matching functions for primaries of 650, 530 and 460

If a complex colour containing more than one wavelength component needs to be reconstructed, then all the specific wavelengths in that complex colour will each have their own coefficients. The complex colour then may be reconstructed using these primaries by simply adding up the coefficients of the primaries needed to reconstruct each wavelength component in that colour.

Thus, if the given colour contained two wavelength components, say λ_{1} and λ_{2} , then by looking up the colour matching functions, the match for each wavelength component can be read off as:

$$\lambda_{1} = r_{1}(\text{Red primary}) + g_{1}(\text{Green primary}) + b_{1}(\text{Blue primary})$$

$$\lambda_{2} = r_{2}(\text{Red primary}) + g_{2}(\text{Green primary}) + b_{2}(\text{Blue primary})$$

where the r , g and b are the amounts of the primaries to match that particular λ . So that, if λ_{1} was our above example of 600 then r_{1} would be 3.33, g_{1} would be 0.27 and b_{1} would be 0.

(Recall, to match 600:

$$600 = 3.33 (\text{Red Primary}) + 0.27 (\text{Green Primary}) + 0 (\text{Blue Primary})$$

Now to reconstruct this complex broadband colour containing both these wavelengths, you simply add r_{1} , g_{1} and b_{1} to r_{2} , g_{2} and b_{2} to get

$$\begin{aligned} \text{Test colour} &= \lambda_{1} + \lambda_{2} \\ &= (r_{1} + r_{2}) (\text{Red primary}) + (g_{1} + g_{2}) (\text{Green primary}) + (b_{1} + b_{2}) (\text{Blue primary}) \end{aligned}$$

If a more complex colour containing many wavelength components, say λ_1 to λ_n , needed to be reconstructed then, to match this complex, multi-wavelength colour, the coefficients of each wavelength component of the colour are added and the final quantity of the red primary, the green primary and the blue primary is obtained. The final quantities of each primary are known as the *tristimulus values*. So, if a colour C is composed of several λ s:

$$C = \lambda_1 + \lambda_2 + \dots + \lambda_n$$

Then to reproduce this with the Red, Green and Blue primaries would require

$$\begin{aligned} C &= \lambda_1 + \lambda_2 + \dots + \lambda_n \\ &= (r_1 + r_2 + \dots + r_n) (\text{Red primary}) + (g_1 + g_2 + \dots + g_n) (\text{Green primary}) + (b_1 + b_2 + \dots + b_n) (\text{Blue primary}) \\ &= R (\text{Red primary}) + G (\text{Green primary}) + B (\text{Blue primary}) \end{aligned}$$

Where R is the sum of the coefficients for the red primary, G the sum for the green primary and B the sum for the blue primary. The values R, G, B are then tristimulus values for this particular compound colour from these specific primaries.

As an example, taking the two earlier colours of 490 and 600. Recall that each colour was matched by the following proportions of the red, green and blue primaries:

$$600 = 3.33 (\text{Red primary}) + 0.27 (\text{Green primary}) + 0 (\text{Blue primary})$$

$$490 = -0.25 (\text{Red primary}) + 0.25 (\text{Green primary}) + 0.3 (\text{Blue primary})$$

If the colour C created by the mix of these two wavelengths is to be matched by the primaries, then

$$\begin{aligned} C = 600 + 490 &= (3.33 - 0.25) (\text{Red primary}) + (0.27 + 0.25) (\text{reen primary}) + (0 + 0.3) (\text{Blue primary}) \\ &= 3.08 (\text{Red primary}) + 0.52 (\text{Green primary}) + 0.3 (\text{Blue primary}) \end{aligned}$$

and the tristimulus values for this resulting colour is: (R, G, B) = (3.08, 0.52 and 0.3)

Note that the tristimulus values are the sum of many coefficients, one for each wavelength component of the complex colour. Thus, while some wavelength components may have negative coefficients, some may have positive coefficients. If the total sum is positive, the tristimulus value will be positive and the complex colour can be matched.

The Colour Triangle and the Chromaticity Diagram

While finding the tristimulus values for any colour is possible by using a combination of the eye sensitivity curves and the colour matching functions, this is tedious and time-consuming. A more convenient way of determining the tristimulus values is by means of a colour triangle and a chromaticity chart. These two diagrams create a geometrical (and possibly more intuitive) representation of colour from which exact numbers required to match any colour with a set of primary colours can be obtained by reading off the coordinates of that colour.

In order to specify a particular colour geometrically, using a specific set of primaries, a coordinate system is set up and a triangle is created wherein all the colours can be represented. The coordinates along each axis are values of the red primary, the green primary and the blue primary. The orientation of the coordinate system is arbitrary, the axes don't have to be at right angles to each other, and the scale of each axis is arbitrary. However, a convenient scale is such that the unit amounts along each axis are the primaries themselves. Thus, on this scale, $R=1$ represents the red primary and $G=1$ and $B=1$ represent the green and blue primaries respectively. The orientation of the axes can also be such that the triangle made by the three points

$R=1, G=0, B=0$ (Point representing the red primary)
 $R=0, G=1, B=0$ (Point representing the green primary)
 $R=0, G=0, B=1$ (Point representing the blue primary)

is an equilateral triangle. In such an orientation and scale, the middle of the triangle is the white point where $R=G=B=1$. Furthermore, any point on the triangle is such that the sum of the coordinates is always 1. This is illustrated in fig 5 (taken from "Colour in Business, Science, and Industry" by Judd Wyszecki).

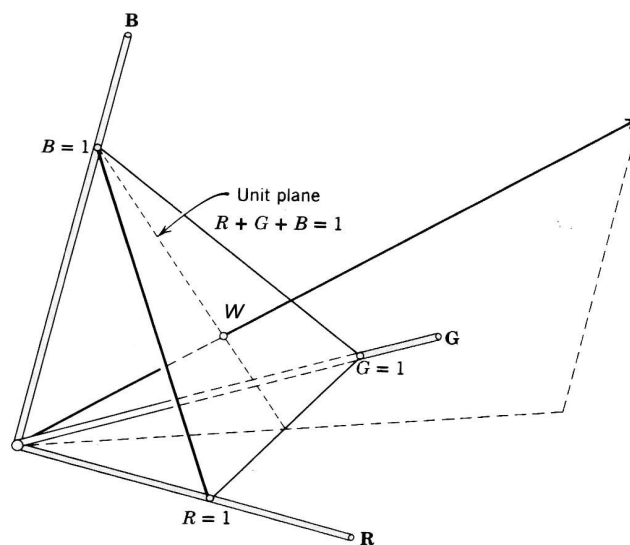


Fig 5 The Colour Triangle. The arrow through the center of the triangle goes through the coordinates of the white point, i.e. at this point $R = G = B = 1$

Any other colour, C, besides white can be represented in this coordinate system by its tristimulus values (R, G, B). Recall that the tristimulus values are the sum of each of the primaries needed to match each wavelength that exists in the colour C. In general, the point representing the tristimulus values for C, i.e. the coordinates R, G and B, will not lie on the triangle, but may be above it or below it, since the sum of the tristimulus values will not add to 1. So, to situate the representation of the colours on the actual triangle, a line is drawn from the origin to the coordinates R, G, B. This line, or its extension if it should fall short of the triangle, cuts the triangle at a specific point. This point, where the line cuts the triangle, is now denoted by r, g, b (small letters). So, any colour can be specified on the actual triangle by taking the intersection of the line representing C, given by its tristimulus values with the triangle. Thus all colours can now be mapped onto the triangle and specified by the point of intersection of the tristimulus values and the triangle. These points, (r, g, b), are called the *chromaticity coordinates* of the colour, with respect to the primaries. Since the triangle is defined by $R + G + B = 1$, any coordinates on the triangle itself will sum to 1 and so the sum of the chromaticity coordinates also sums to 1: $r + g + b = 1$.

This is illustrated in fig 6 and 7 (taken from “Colour in Business, Science, and Industry” by Judd Wyszecki) for the colour C.

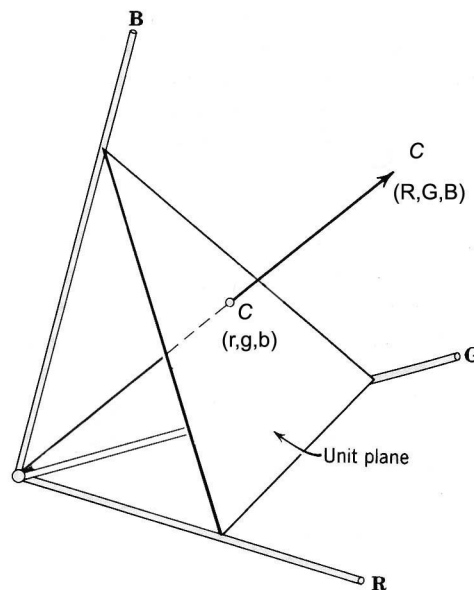


Fig 6 The colour triangle with a colour C, its tristimulus values and its chromaticity coordinates.

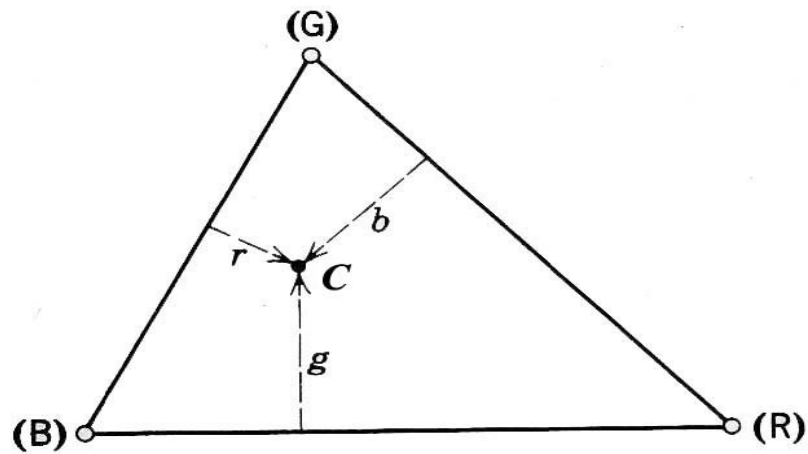


Fig 7 Chromaticity coordinates for C on just the triangle, as seen straight-on.

The chromaticity coordinates r , g , b are related to the tristimulus coordinates R , G , B by

$$r = R / (R + G + B)$$

$$g = G / (R + G + B)$$

$$b = B / (R + G + B)$$

and, again

$$r + g + b = 1$$

Another variation of the orientation of axes may be such that the triangle is not equilateral, but right-angled. This makes it easier and more intuitive to read off the chromaticity coordinates. In this orientation, we choose two sides of the triangle, which are now at right angles to each other, to form a new set of axes for the red chromaticity coordinate and the green chromaticity coordinate. The blue chromaticity coordinate is derived by $b = 1 - r - g$.

The resulting diagram is known as the *chromaticity chart*. Fig 8 (adapted from “The Reproduction of Colour” by R.W.G. Hunt) shows the chromaticity chart resulting from the primaries used in the examples above: 650, 530 and 460

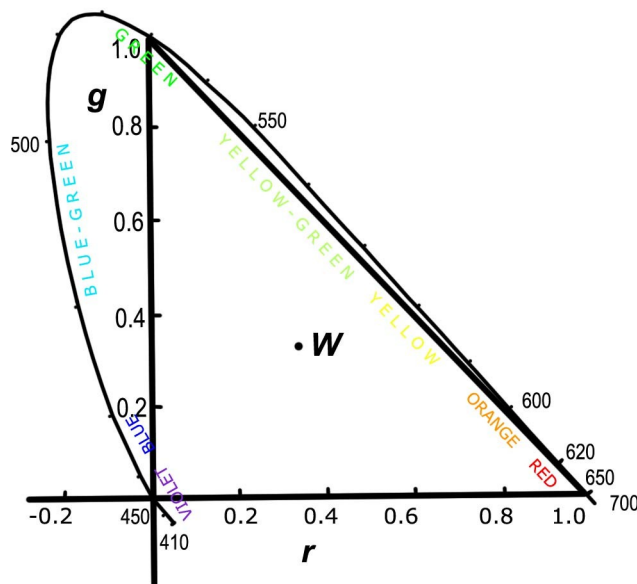


Fig 8 Chromaticity diagram for the primaries 650, 530 and 460.

Note the point $(r,g) = (1,0)$ represents the red primary of 650 and the $(r,g) = (0,1)$ represents the green of 530.

Note also that some colours are outside the triangle, in the second and fourth quadrants. All colours inside the triangle have positive coefficients and are known as being *inside the gamut*. These can be reproduced by this particular set of primaries. All colours outside the triangle are known as being *outside the gamut* and cannot be reproduced (at least, not without artificially adding the primaries to the colour to be matched as described in section on negative coefficients).

1931 CIE

In order to standardise the description of colour, the CIE used a series of experimental observations by W. D. Wright and J. Guild using 17 subjects with normal colour vision and by K. S. Gibson and E. P. T. Tyndall with about a hundred subjects. These subjects matched a series of colours using the primaries 700, 546.1 and 435.8., denoted by R_0 , G_0 and B_0 . The corresponding chromaticity coordinates are denoted by \bar{r} , \bar{g} and \bar{b} (the actual coordinates are denoted by having a bar across the top of the letters). From these observations, the colour-matching functions were created as in fig 9 (From “The Reproduction of Colour” by R.W.G. Hunt).

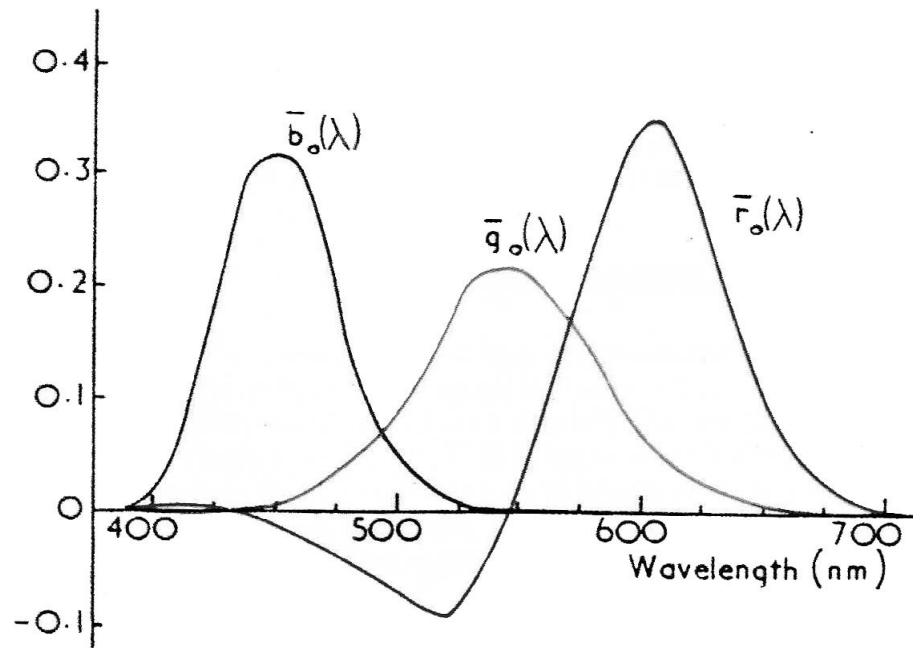


Fig 9 Color matching functions for the W. D. Wright and J. Guild primaries of 700, 546.1 and 435.8.

As can be seen, with these primaries, there is a significant number of colours, from about 425 to about 550, which cannot be matched due to the negative red coefficient. In order to overcome this and other problems, the CIE transformed the primaries to a new set of primaries denoted by X, Y and Z. These new primaries are not actually real colours but are mathematical numbers created in order to ensure that there are no negative coefficients. The transformations between R_0 , G_0 and B_0 . And X, Y, Z are given by:

$$\begin{aligned}
 X &= 0.49 R_0 + 0.31 G_0 + 0.20 B_0 \\
 Y &= 0.1769 R_0 + 0.8124 G_0 + 0.1063 B_0 \\
 Z &= 0.00 R_0 + 0.01 G_0 + 0.99 B_0
 \end{aligned}$$

The chromaticity chart for the primaries used in the experimental study (700, 546.1 and 435.8) is shown in fig 10 (From "The Reproduction of Colour" by R.W.G. Hunt), along with the positions of the hypothetical new primaries of X, Y and Z.

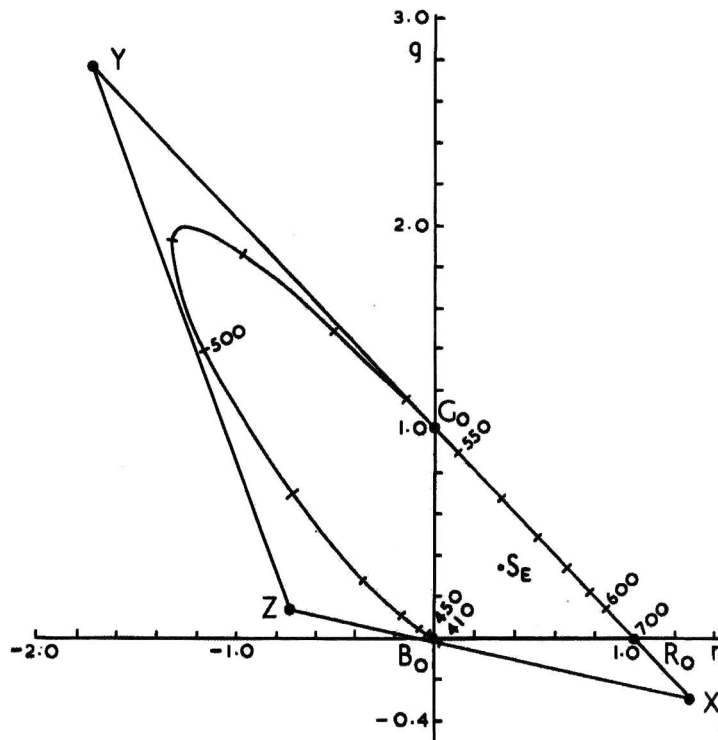


Fig 10 Chromaticity chart for W. D. Wright and J. Guild primaries, with the positions of the 1931 X, Y, Z primaries.

From these new primaries, a new set of chromaticity coordinates, \bar{x} , \bar{y} and \bar{z} (again, the bar is above the letter in the literature) are derived from

$$\bar{x} = X/(X + Y + Z)$$

$$\bar{y} = Y/(X + Y + Z)$$

$$\bar{z} = Z/(X + Y + Z)$$

And, from these values, new colour matching function curves are drawn, as in fig 11

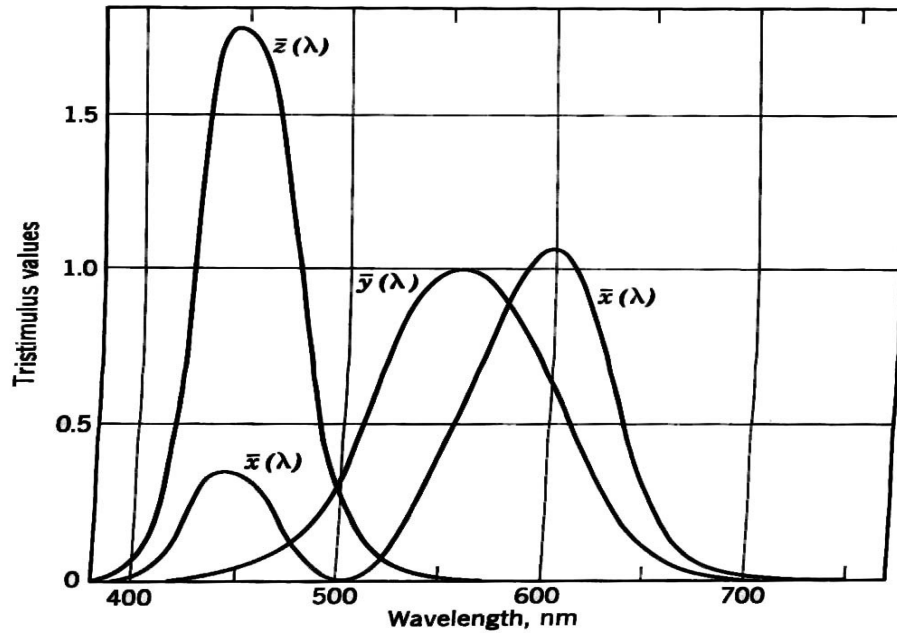


Fig 11 Colour matching functions using X, Y and Z primaries

It can now be seen that there are no negative coefficients. The new chromaticity chart, derived from these colour matching functions is shown in fig 12. It can now be seen that the new chart has the entire gamut within the triangle. However, note that to actually match a given colour by using the coordinates in the CIE 1931 chart, a transformation must be made from the X, Y, Z coordinates used to make the CIE chart, to the actual, real primaries to be used to match the real colour. When these transformations are carried out, negative coefficients may still appear.

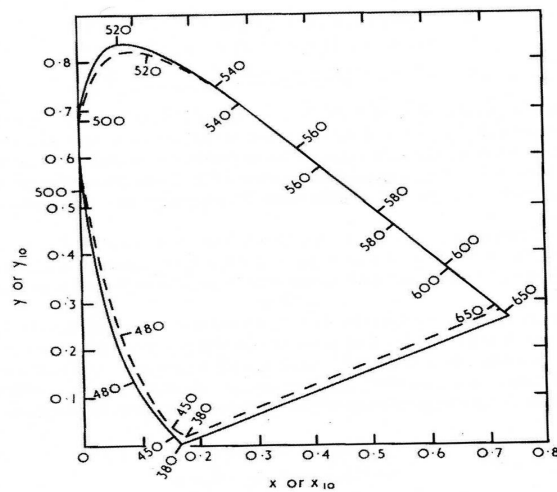


Fig 12 Chromaticity chart from the new primaries X, Y, Z